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If $r=n-1$, the numerator and the denominator are of equal degree, and, being composed of the same factors, the fraction equals 1.

If $r=n$, the degree of the numerator is one greater than that of the denominator. Hence, besides the factors common to both, there must be in the numerator one other factor of the first degree. Since this factor must be symmetrical with reference to a_1, a_2, a_3 , etc., it is $a_1 + a_2 + a_3 + \dots a_n$.

This last is, therefore, the value of the fraction, the numerical coefficient independent of a_1, a_2, a_3 , etc., evidently being unity.

Also solved by C. W. M. BLACK.

82. Proposed by B. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and F. P. MATZ, D. Sc., Ph.D. Mechanicsburg, Pa.

$$\left. \begin{aligned} y^2 + yz + z^2 &= a^2 \\ z^2 + zx + x^2 &= b^2 \\ x^2 + xy + y^2 &= c^2 \end{aligned} \right\} \text{find } x, y, \text{ and } z.$$

[C. Smith's *Treatise on Algebra*, Ex. 31, page 172.]

I. Solution by P. S. BERG, A. M., Principal of Schools, Larimore, N. D.

$$\text{From (1) + (2) + (3), } 2(y^2 + z^2 + x^2) + yz + zx + xy = a^2 + b^2 + c^2 \dots\dots\dots (4).$$

Squaring (4)

$$4(y^2 + z^2 + x^2)^2 + 4(y^2 + z^2 + x^2)(yz + zx + xy) + (yz + zx + xy)^2 = (a^2 + b^2 + c^2)^2 \dots (5).$$

From $2(1)^2 + 2(2)^2 + 2(3)$

$$4(y^2 + z^2 + x^2)^2 + 4(y^2 + z^2 + x^2)(yz + zx + xy) - 2(yz + zx + xy)^2 = 2(a^4 + b^4 + c^4) \dots (6).$$

From $\sqrt{\{(5) - (6)\}/3}$

$$yz + zx + xy = \pm \sqrt{\{ \frac{1}{3} [(2a^2b^2 + 2b^2c^2 + 2a^2c^2) - (a^4 + b^4 + c^4)] \}} \dots\dots\dots (7).$$

Put second member = m ; then from $\sqrt{[6(7) + 2(4)]}$

$$2(y + z + x) = \pm \sqrt{[2(a^2 + b^2 + c^2) + 6m]} \dots\dots\dots (8).$$

$$\text{From (4) + (7) - 2(2) } 2y(y + z + x) = a^2 - b^2 + c^2 + m \dots\dots\dots (9).$$

From (9) $\div$ (8) and restoring m

$$y = \frac{a^2 - b^2 + c^2 \pm \frac{1}{3} \sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}{\pm \sqrt{[2(a^2 + b^2 + c^2) \pm 2\sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}}}.$$

$$\text{Similarly, } z = \frac{a^2 + b^2 - c^2 \pm \frac{1}{3} \sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}{\pm \sqrt{[2(a^2 + b^2 + c^2) \pm 2\sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}}},$$

$$\text{and } x = \frac{b^2 + c^2 - a^2 \pm \frac{1}{3} \sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}{\pm \sqrt{[2(a^2 + b^2 + c^2) \pm 2\sqrt{[12a^2b^2 - 3(a^2 + b^2 - c^2)^2]}}}}.$$

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Subtracting (1) from (2) we get $(x-y)(x+y+z) = b^2 - a^2$, or putting $x+y+z=s$, $(x-y)s = b^2 - a^2 \dots\dots (4)$, and subtracting (1) from (3), we thus get $(x-z)s = c^2 - a^2 \dots\dots (5)$. From (4) and (5) we obtain $y = (sx + a^2 - b^2)/s \dots\dots (6)$, and $z = (sx + a^2 - c^2)/s \dots\dots (7)$. Adding x to both members of (6) and (7), we have $x+y+z = (3sx + 2a^2 - b^2 - c^2)/s$, or $s^2 = 3sx + 2a^2 - b^2 - c^2 \dots\dots (8)$, whence

$x=(s^2+b^2+c^2-2a^2)/3s\dots\dots(9)$, and then by (6) and (7), $y=(s^2+a^2-2b^2+c^2)/3s\dots\dots(10)$; $z=(s^2+a^2+b^2-2c^2)/3s\dots\dots(11)$.

Substituting in (3), expanding, and arranging, we obtain,

$$s^4-(a^2+b^2+c^2)s^2=a^2b^2+a^2c^2+b^2c^2-a^4-b^4-c^4\dots\dots(12),$$

whence $s^2=\frac{1}{2}[a^2+b^2+c^2\pm\sqrt{3}\sqrt{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}]$, or putting the irrational part= S , so that $S=\sqrt{3}\sqrt{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}$(13). $s^2=\frac{1}{2}(a^2+b^2+c^2\pm S)\dots\dots(14)$.

Substituting now in (9), (10), (11), we obtain

$$x=[3(b^2+c^2-a^2)\pm S]/6s;$$

$$y=[3(a^2+c^2-b^2)\pm S]/6s;$$

$$z=[3(a^2+b^2-c^2)\pm S]/6s.$$

(13), (14), and (15) suffice to determine the values of x , y , and z .

III. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

$$(1)-(2) \text{ gives } (y-x)(x+y+z)=a^2-b^2\dots\dots(4).$$

$$(1)-(3) \text{ gives } (z-x)(x+y+z)=a^2-c^2\dots\dots(5).$$

$$(4)\div(5) \text{ gives } y=[(b^2-c^2)x+(a^2-b^2)z]/[a^2-c^2]\dots\dots(6).$$

$$(6) \text{ in } (3) \text{ gives } (a^4+b^4+3c^4+a^2b^2-3a^2c^2-3b^2c^2)x^2+(a^4+a^2b^2+3b^2c^2-2b^4-3a^2c^2)xz+(a^4-2a^2b^2+b^4)z^2=a^4c^2-2a^2c^4+c^6\dots\dots(7).$$

Let $z=vx$ in (2) and (7), and divide (7) by (2),

$$(a^4b^2-2a^2b^4+b^6-a^4c^2+2a^2c^4-c^6)v^2+(a^4b^2-2b^6+3b^4c^2+a^2b^4-3a^2b^2c^2-a^4c^2+2a^2c^4-c^6)v=a^4c^2+2a^2c^4-c^6-a^4b^2-b^6-3b^2c^4-a^2b^4+3a^2b^2c^2-3b^4c^2,$$

or $v^2+2Av=B$, suppose. $\therefore v=\pm\sqrt{A^2+B}-A$.

$$x=\frac{b}{\sqrt{(1+v+v^2)}}=\frac{b^2+c^2-a^2\pm\frac{1}{2}\sqrt{[12a^2b^2-3(a^2+b^2-c^2)^2]}}{\sqrt{\{2(a^2+b^2+c^2)\pm2\sqrt{[12a^2b^2-3(a^2+b^2-c^2)^2]}\}}}.$$

$$\therefore x=(b^2+c^2-a^2\pm C)/D, \text{ suppose.}$$

$$y=(a^2+c^2-b^2\pm C)/D.$$

$$z=(a^2+b^2-c^2\pm C)/D.$$

Prof. Cooper D. Schmitt remarks that nine different solutions of this problem can be found in the *Mathematical Magazine*, Vol. II, Nos. 8 and 10, on pages 141 and 193.

Prof. Zerr sent three solutions.